PERTH MODERN SCHOOL
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INDEPENDENT PUBLIC SCHOOL

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST
UNIT 1

Section One:
Calculator-free

Your Name________________________________

Your Teacher's Name______________________________

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Max	Question	Mark	Max
1		5	5		8
2		7	6		8
3		7	7		5
4		10			

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	33
Section Two: Calculator-assumed	11	11	100	100	67
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 11 Information Handbook 2021*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free **(50 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 {1.1.3, 1.1.8} **(5 marks)**

(a) Simplify ${}^{n+3}C_{n+1}$ (2 marks)

Solution
$${}^{n+3}C_{n+1} = \frac{(n+3)!}{(n+1)!\,(n+3-n-1)!}$$ $$= \frac{(n+3)(n+2)(n+1)!}{(n+1)!\,2!}$$ $$= \frac{(n+3)(n+2)}{2}$$
Specific behaviours
✓ uses ${}^{n}C_r = \frac{n!}{(n-r)!r!}$ ✓ eliminates factorials and simplifies

(b) Find the value of n, if $9 \times {}^{n}P_3 = 10 \times {}^{n-1}P_3$ (3 marks)

Solution
$$\frac{9 \times n!}{(n-3)!} = \frac{10(n-1)!}{(n-4)!}$$ $$\frac{9n(n-1)(n-2)(n-3)!}{(n-3)!} = \frac{10(n-1)(n-2)(n-3)(n-4)!}{(n-4)!}$$ $$9n(n-1)(n-2) = 10(n-1)(n-2)(n-3)$$ $$9n = 10(n-3)$$ $$n = 30$$
Specific behaviours
✓ uses ${}^{n}P_r = \frac{n!}{(n-r)!}$ ✓ eliminates factorials and fully simplifies   ✓ solves correctly for n

Question 2 {1.3.1, 2.3.1} **(7 marks)**

(a) Complete the table below. (3 marks)

Statement	*If* $x = 4$, *then* $x^2 = 16$.	*True*
Inverse of Statement	If $x \neq 4$, then $x^2 \neq 16$.	False
Converse of Statement	If $x^2 = 16$, then $x = 4$.	False
Contrapositive of Statement	If $x^2 \neq 16$, then $x \neq 4$.	True

Solution
See table.
Specific behaviours
✓ correct inverse and states false (R/W) ✓ correct converse and states false (R/W) ✓ correct contrapositive and states true (R/W)

(b) Let $n \in Z$, prove that if $n + 2$ is even, then $n - 1$ is odd. (4 marks)

Solution

If $n + 2$ is even, then $n + 2 = 2k\ for\ some\ k \in Z$, and so

$$n - 1 = n + 2 - 3$$
$$= 2k - 3$$
$$= 2k - 4 + 1$$
$$= 2(k - 2) + 1$$

Hence $n - 1$ is odd.

Specific behaviours

✓ expresses $n + 2$ as $2k$ and states $k \in Z$

✓ expresses $n - 1$ as $n + 2 - 3$ or n as $2k - 2$

✓ expresses $n - 1$ in terms of $2k$

✓ factorises $2k - 4 + 1$ or $2k - 2 - 1$

Question 3 {1.2.2, 1.2.3, 1.2.5} **(7 marks)**

Vector $\begin{bmatrix} a \\ a+b \end{bmatrix}$ has a magnitude of 10 and is parallel to vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Find all possible values of a and b.

Solution
$\begin{bmatrix} a \\ a+b \end{bmatrix} = \lambda \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $where\ \lambda\ is\ a\ non-zero\ real\ number$ $a = 2\lambda \Rightarrow \lambda = \frac{a}{2}$ $a + b = 4\lambda$ $= 2a$ $\Rightarrow a = b$ $\sqrt{a^2 + (a+b)^2} = 10$ $a^2 + 4a^2 = 100$ $5a^2 = 100$ $a^2 = 20$ $a = \pm 2\sqrt{5}$ $\therefore a = b = 2\sqrt{5}$ *or* $a = b = -2\sqrt{5}$
Specific behaviours
✓ writes $\begin{bmatrix} a \\ a+b \end{bmatrix}$ as a scalar multiple of $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ ✓ expresses the scalar λ in terms of a ✓ substitutes λ ✓ states $a = b$ ✓ uses the magnitude of 10   ✓ determines one sets of values for $a\ and\ b$ ✓ determines two sets of values for $a\ and\ b$ *Alternative methods: unit vector or dot product*

Question 4 {1.2.6, 1.2.7, 1.2.8, 1.2.9} **(10 marks)**

Given the vectors $\boldsymbol{p} = 10\boldsymbol{i} + 6\boldsymbol{j}, \boldsymbol{q} = 4\boldsymbol{i} + x\boldsymbol{j}, \boldsymbol{r} = \frac{2}{3}\boldsymbol{i} + y\boldsymbol{j}$ and $\boldsymbol{s} = -11\boldsymbol{i} - z\boldsymbol{j}$, find the exact values of x, y and z for which:

(a) $\boldsymbol{p} + \boldsymbol{q}$ is parallel to $\boldsymbol{p} - \boldsymbol{q}$, (3 marks)

Solution

If $\boldsymbol{p} + \boldsymbol{q}$ is parallel to $\boldsymbol{p} - \boldsymbol{q}$, there exists a non-zero real number k such that

$$k(\boldsymbol{p} + \boldsymbol{q}) = \boldsymbol{p} - \boldsymbol{q}$$

$$k(14\boldsymbol{i} + (6 + x)\boldsymbol{j} = 6\boldsymbol{i} + (6 - x)\boldsymbol{j}$$

$$\therefore 14k = 6 \ \Rightarrow k = \frac{3}{7}$$

$$\therefore k(6 + x) = (6 - x)$$

$$\frac{3}{7}(6 + x) = (6 - x)$$

$$\Rightarrow x = \frac{12}{5} \text{ or } 2.4$$

Specific behaviours

✓ writes $\boldsymbol{p} + \boldsymbol{q}$ as a scalar multiple of $\boldsymbol{p} - \boldsymbol{q}$ and substitutes $\boldsymbol{p}\ and\ \boldsymbol{q}$

✓ solves for k

✓ solves for x

(b) $\boldsymbol{r}$ is a unit vector, (3 marks)

Solution

$$|\boldsymbol{r}| = \sqrt{\left(\frac{2}{3}\right)^2 + y^2} = 1$$

$$\frac{4}{9} + y^2 = 1$$

$$y^2 = \frac{5}{9}$$

$$\therefore y = \pm\frac{\sqrt{5}}{3}$$

Specific behaviours

✓ uses the magnitude of 1

✓ one correct value for y

✓ two correct values for y

(c) the resultant of $\boldsymbol{p}$ and $\boldsymbol{s}$ has magnitude 5 units. (4 marks)

Solution

$$\boldsymbol{p} + \boldsymbol{s} = -\boldsymbol{i} + (6 - z)\boldsymbol{j}$$

$$|\boldsymbol{p} + \boldsymbol{s}| = \sqrt{(-1)^2 + (6 - z)^2} = 5$$

$$1 + (6 - z)^2 = 25$$

$$z = 6 \pm 2\sqrt{6}$$

Specific behaviours

✓ expresses the resultant vector in component form

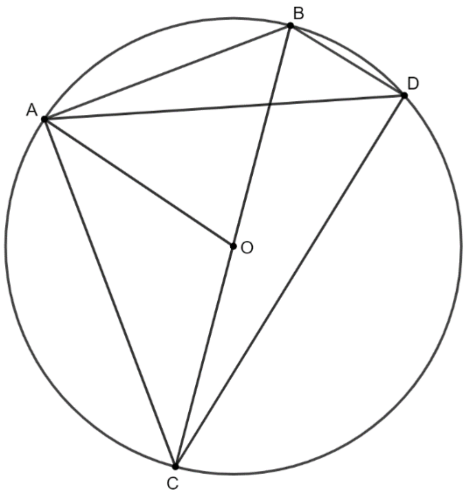
✓ uses the magnitude of 5

✓ one correct value for z

✓ two correct values for z

Question 5 {1.3.6, 1.3.7, 1.3.8, 1.3.9} **(8 marks)**

Point O is the centre of the circle below. Given A, B, C, D are points on the circle, $\angle AOB = 60°$ and $\angle CBD = 70°$, complete the table below.

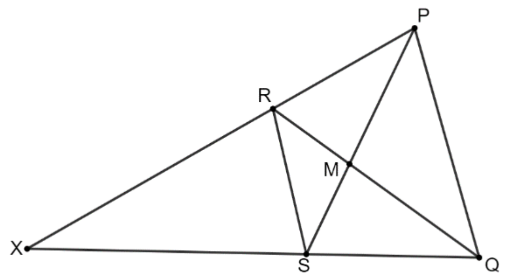


Angle	Size	Reason
$\angle BCD$	$20°$	Angle in a semicircle
$\angle ADB$	$30°$	Angle at the circumference $= \frac{1}{2} \times$ angle at the centre
$\angle ACD$	$50°$	Angles in the same segment
$\angle ABD$	$130°$	Cyclic quadrilateral

Solution
See table.
Specific behaviours
✓✓ states correct size of $\angle BCD$ with reason(s) ✓✓ states correct size of $\angle ADB$ with reason(s) ✓✓ states correct size of $\angle ACD$ with reason(s) ✓✓ states correct size of $\angle ABD$ with reason(s)

Question 6 {1.3.10, 1.3.12} **(8 marks)**

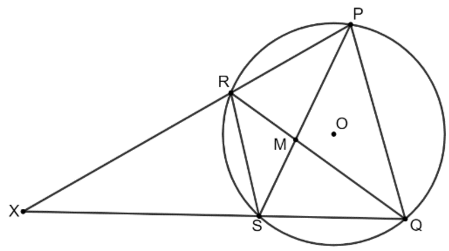
In ΔXPQ(diagram is not drawn to scale), R and S are points on XP and XQ, respectively. PS and QR intersects at M. Given that $\angle PMR = 100°$, $\angle XPS = 20°$ and $\angle PSQ = 60°$,



(a) Prove that $XP \cdot XR = XQ \cdot XS$. (5 marks)

$\angle QMS = \angle PMR = 100°$ (vertically opposite) $\angle RQX = 180° - 100° - 60° = 20°$ (angle sum of Δ) For ΔXPS and ΔXQR, $\angle RQX = \angle XPS = 20°$ $\angle PXS = \angle QXR$ (common) $\therefore \Delta XPS \sim \Delta XQR\ (AA)$ $\frac{XP}{XQ} = \frac{XS}{XR}$ $XP \cdot XR = XQ \cdot XS$
Specific behaviours
✓ states correct size of $\angle QMS$ with reason ✓ states correct size of $\angle RQX$ with reason ✓ proves $\Delta XPS \sim \Delta XQR$ with reason ✓ uses corresponding sides of $\Delta XPS\ and\ \Delta XQR$ ✓ obtains $XP \cdot XR = XQ \cdot XS$

(b) Part (a) implies that P, Q, S and R lie on a circle where point O is the centre of the circle. Given that $PR = QS$, prove that $PQ \parallel RS$. (3 marks)



Solution
$\because PR = QS$ $\therefore \angle ROP = \angle SOQ$ (chords of equal length subtend equal angles at the centre) $\therefore \angle RSP = \angle SPQ$ (angle at the circumference = $\frac{1}{2} \times$ angle at the centre) $\therefore PQ \parallel RS\ (alternate\ angles\ are\ equal)$
Specific behaviours
✓ uses $PR = QS$ ✓ gives reason (chords of equal length subtend equal angles) ✓ states alternate angles are equal to prove parallel lines

Question 7 {1.3.2} **(5 marks)**

Suppose that a and b are positive integers such that $\sqrt{a}$ is irrational. Use the method of 'proof by contradiction' to prove that $\left(\sqrt{a}+b\right)^2$ is not an integer.

Solution
Assume that $\left(\sqrt{a}+b\right)^2 = k$ where k is an integer. Then $$a + 2b\sqrt{a} + b^2 = k$$ $$2b\sqrt{a} = k - a - b^2$$ $$\sqrt{a} = \frac{k-a-b^2}{2b}$$ But since k, a and b are integers, $\frac{k-a-b^2}{2b}$ is rational, which contradicts the fact that $\sqrt{a}$ is irrational. Hence $\left(\sqrt{a}+b\right)^2$ is not an integer.
Specific behaviours
✓ assumes that $\left(\sqrt{a}+b\right)^2$ is an integer ✓ expends $\left(\sqrt{a}+b\right)^2$ ✓ expresses $\sqrt{a}$ in terms of k, a and b ✓ argues that $\sqrt{a}$ is therefore rational ✓ notes contradiction and concludes that $\left(\sqrt{a}+b\right)^2$ is not an integer

Additional working space

Question number: ________

Additional working space

Question number: ________

Additional working space

Question number: ________